35. $34.2 + (-34.2) = 0$

36. $(4 \cdot 5) \cdot 7 = 4 \cdot (5 \cdot 7)$

37. $\frac{0}{17}$

38. $\frac{-8}{0}$

39. $\frac{2}{3} \cdot \left(-\frac{12}{43}\right) \cdot \frac{3}{2} = \frac{2}{3} \cdot \frac{3}{2} \cdot \left(-\frac{12}{43}\right)$

40. $\frac{5}{12} \cdot \frac{12}{5} = 1$

41. $5.23 + 4.98 + (-5.23) = 5.23 + (-5.23) + 4.98$

42. $16.4 \cdot 0 = 0$

43. $\frac{21}{0}$

44. $\frac{0}{106}$

Mixed Practice

In Problems 45–64, evaluate each expression by using the properties of real numbers.

45. $54 + 29 + (-54)$

46. $46 + 59 + (-46)$

47. $\frac{9}{5} \cdot \frac{5}{9} \cdot 18$

48. $\frac{4}{9} \cdot \frac{9}{4} \cdot 28$

49. $-25 \cdot 13 \cdot \frac{1}{5}$

50. $36 \cdot (-12) \cdot \frac{1}{6}$

51. $347 + 456 + (-456)$

52. $593 + 306 + (-306)$

53. $\frac{9}{2} \cdot \left(-\frac{10}{3}\right) \cdot 6$

54. $\frac{13}{2} \cdot \frac{8}{39} \cdot \frac{39}{4}$

55. $\frac{7}{0}$

56. $\frac{0}{100}$

57. $100(-34)(0.01)$

58. $4000(0.5)(0.001)$

59. $569.003 \cdot 0$

60. $104 \cdot \frac{1}{104}$

61. $\frac{45}{3902} + \left(-\frac{45}{3902}\right)$

62. $30 \cdot \frac{4}{4}$

63. $-\frac{5}{44} \cdot \frac{80}{3} \cdot \frac{11}{5}$

64. $\frac{7}{48} \cdot \left(-\frac{21}{4}\right) \cdot \frac{12}{7}$

Applying the Concepts

65. Balancing the Checkbook Alberto's checking account balance at the start of the month was \$321.03. During the month, he wrote checks for \$32.84, \$85.03, and \$120.56. He also deposited a check for \$120.56. What is Alberto's balance at the end of the month?

66. Stock Price Before the opening bell on Monday, a certain stock was priced at \$32.04. On Monday the stock was up \$0.54, on Tuesday it was down \$0.32, and on Wednesday it was down \$0.54. What was the closing price of the stock on Wednesday?

Extending the Concepts

In Problems 67–70, insert parentheses to make the statement true.

67. $-3 - 4 - 10 = 3$

68. $-6 - 4 + 10 = -20$

69. $-15 + 10 - 4 - 8 = -1$

70. $25 - 6 - 10 - 1 = 28$

71. Convert 30 miles per hour to feet per second. (*Note*: 1 mile = 5280 feet)

72. Convert 40 miles per hour to feet per second. (*Note*: 1 mile = 5280 feet)

Explaining the Concepts

73. In your own words, explain why 0 does not have a multiplicative inverse.

74. Why does $2(4 \cdot 5)$ not equal $(2 \cdot 4) \cdot (2 \cdot 5)$?

75. Why does $\frac{0}{4} = 0$? Why is $\frac{4}{0}$ undefined?

76. How is the Identity Property of Addition related to the Additive Inverse Property?

77. How is the Identity Property of Multiplication related to the Multiplicative Inverse Property?

78. Let a be a real number such that $a > 0$, and let b represent a real number such that $b < 0$. Indicate whether each of the following statements are true or false, and explain your decision using actual values for a and b.

(a) $a > -a$

(b) $b > -b$

(c) $a + (-a) = 0$

(d) $-b + b = 0$

(e) $a - (-b) > 0$

(f) $ab < 0$

(g) $-ab > 0$

(h) $\frac{-a}{-b} > 0$

1.7 Exponents and the Order of Operations

Objectives

1. Evaluate Exponential Expressions
2. Apply the Rules for Order of Operations

Are You Ready for This Section?

Before getting started, take this readiness quiz. If you get a problem wrong, go back to the section cited and review the material.

R1. Find the sum: $9 + (-19)$ [Section 1.4, pp. 28–30]

R2. Find the difference: $28 - (-7)$ [Section 1.4, pp. 31–32]

R3. Find the product: $-7 \cdot \frac{8}{3} \cdot 36$ [Section 1.5, pp. 37–38]

R4. Find the quotient: $\frac{100}{-15}$ [Section 1.4, pp. 34–35]

1 Evaluate Exponential Expressions

If we wanted to multiply 2 eight times, we would write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. That's a lot of writing! To reduce the amount of writing needed to show repeated multiplication, we use **exponential notation**, where we write $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^8. In 2^8, 2 is called the **base** and 8 is called the **exponent**.

Exponential Notation

If n is a natural number and a is a real number, then

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$$

where a is the **base** and n is the **exponent** or **power.** The exponent tells the number of times the base is used as a factor.

An expression written in the form a^n is said to be in **exponential form.** The expression 6^2 is read "six squared," 8^3 is read "eight cubed," and the expression 11^4 is read "eleven to the fourth power." In general, we read a^n as "a to the nth power."

EXAMPLE 1 Writing a Numerical Expression in Exponential Form

Write each expression in exponential form.

(a) $5 \cdot 5 \cdot 5$ **(b)** $(-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4)$

Solution

(a) The expression $5 \cdot 5 \cdot 5$ contains three factors of 5, so $5 \cdot 5 \cdot 5 = 5^3$.

(b) The expression $(-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4)$ contains six factors of -4, so $(-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4) \cdot (-4) = (-4)^6$.

Quick ✓

1. In the expression 3^6, 3 is the ____ and 6 is the ________ or ______.

In Problems 2 and 3, write each expression in exponential form.

2. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$
3. $(-7) \cdot (-7) \cdot (-7) \cdot (-7)$

To evaluate an exponential expression, write the expression in **expanded form.** For example, 2^8 in expanded form is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

Ready?...Answers **R1.** -10 **R2.** 35 **R3.** -672 **R4.** $-\frac{20}{3}$

EXAMPLE 2 **Evaluating an Exponential Expression**

Evaluate each exponential expression:

(a) 6^4 **(b)** $\left(\frac{5}{3}\right)^5$

Solution

(a) $6^4 = 6 \cdot 6 \cdot 6 \cdot 6$
$= 1296$

(b) $\left(\frac{5}{3}\right)^5 = \left(\frac{5}{3}\right)\left(\frac{5}{3}\right)\left(\frac{5}{3}\right)\left(\frac{5}{3}\right)\left(\frac{5}{3}\right)$
$= \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}$
$= \frac{3125}{243}$

EXAMPLE 3 **Evaluating an Exponential Expression—Odd Exponent**

Evaluate each exponential expression:

(a) $(-5)^3$ **(b)** -5^3

Solution

(a) $(-5)^3 = (-5) \cdot (-5) \cdot (-5)$
$= -125$

(b) $-5^3 = -(5 \cdot 5 \cdot 5)$
$= -125$

EXAMPLE 4 **Evaluating an Exponential Expression—Even Exponent**

Evaluate each exponential expression:

(a) $(-5)^4$ **(b)** -5^4

Solution

(a) $(-5)^4 = (-5) \cdot (-5) \cdot (-5) \cdot (-5)$
$= 625$

(b) $-5^4 = -(5 \cdot 5 \cdot 5 \cdot 5)$
$= -625$

Work Smart

There is a difference between $(-5)^4$ and -5^4.

The parentheses in $(-5)^4$ tell us to use four factors of -5. However, in the expression -5^4, we use 5 as a factor four times and then multiply the result by -1. We could also read -5^4 as "Take the opposite of the quantity 5^4."

Quick ✓

In Problems 4–9, evaluate each exponential expression.

4. 2^4 **5.** $(-7)^2$

6. $\left(-\frac{1}{6}\right)^3$ **7.** $(0.9)^2$

8. -2^4 **9.** $(-2)^4$

2 Apply the Rules for Order of Operations

To evaluate $3 \cdot 5 + 4$, do you multiply first and then add to get $15 + 4 = 19$ *or* do you add first and then multiply to get $3 \cdot 9 = 27$?

Because $3 \cdot 5$ is equivalent to $5 + 5 + 5$, we have

$$3 \cdot 5 + 4 = 5 + 5 + 5 + 4$$
$$= 19$$

In Words
Multiply first then add.

Based on this, **whenever addition and multiplication appear in the same expression, always multiply first and then add.**

Because any division problem can be written as a multiplication problem, we divide before adding as well. Also, because any subtraction problem can be written as an addition problem, we always multiply and divide before adding and subtracting.

EXAMPLE 5 Evaluating an Expression Containing Multiplication, Division, and Addition

Evaluate each expression:

(a) $11 + 2 \cdot (-6)$ **(b)** $7 + 12 \div 3 \cdot 5$

Solution

(a) Multiply first: $11 + 2 \cdot (-6) = 11 + (-12)$

Add: $= -1$

(b) Multiply/divide left to right: $7 + 12 \div 3 \cdot 5 = 7 + 4 \cdot 5$

Multiply: $= 7 + 20$

$= 27$

Quick ✓

In Problems 10–13, evaluate each expression.

10. $1 + 7 \cdot 2$ **11.** $-3 \div \left(-\frac{1}{2}\right) + 18$

12. $9 \cdot 4 \div 2 + 5$ **13.** $\frac{15}{2} \div (-5) \cdot 8 - 7$

Parentheses

If we want to add two numbers first and then multiply, we use parentheses and write $(3 + 5) \cdot 4$. In other words, **always evaluate the expression in parentheses first.**

EXAMPLE 6 Finding the Value of an Expression Containing Parentheses

Evaluate each expression:

(a) $(5 + 3) \cdot 2$ **(b)** $\left(\frac{3}{2} - \frac{5}{2}\right)\left(\frac{7}{3} + \frac{2}{3}\right)$

Solution

(a) $(5 + 3) \cdot 2 = 8 \cdot 2$

$= 16$

(b) $\left(\frac{3}{2} - \frac{5}{2}\right)\left(\frac{7}{3} + \frac{2}{3}\right) = \left(-\frac{2}{2}\right)\left(\frac{9}{3}\right)$

$= (-1)(3)$

$= -3$

Quick ✓

In Problems 14–16, evaluate each expression.

14. $8(2 + 3)$ **15.** $(2 - 9) \cdot (5 + 4)$ **16.** $\left(\frac{6}{7} + \frac{8}{7}\right) \cdot \left(\frac{11}{8} + \frac{5}{8}\right)$

Work Smart

The division bar acts like a set of parentheses.

The Division Bar

If an expression contains a division bar, we treat the terms above and below the division bar as if they were in parentheses. For example,

$$\frac{3+5}{9+7} = \frac{(3+5)}{(9+7)} = \frac{8}{16} = \frac{\cancel{8}\cdot 1}{\cancel{8}\cdot 2} = \frac{1}{2}$$

EXAMPLE 7 **Finding the Value of an Expression That Contains a Division Bar**

Evaluate each expression:

(a) $\dfrac{7\cdot 3}{3+9\cdot 2}$ **(b)** $\dfrac{1+7\div\frac{1}{5}}{-6\cdot 2+8}$

Solution

(a) Multiply: $\dfrac{7\cdot 3}{3+9\cdot 2} = \dfrac{21}{3+18}$

Add: $= \dfrac{21}{21}$

$= 1$

(b) Write division as multiplication: $\dfrac{1+7\div\frac{1}{5}}{-6\cdot 2+8} = \dfrac{1+7\cdot 5}{-6\cdot 2+8}$

Multiply: $= \dfrac{1+35}{-12+8}$

Add: $= \dfrac{36}{-4}$

$= \dfrac{9\cdot\cancel{4}}{-1\cdot\cancel{4}}$

$= -9$

Quick ✓

In Problems 17–19, evaluate each expression.

17. $\dfrac{2+5\cdot 6}{-3\cdot 8-4}$ **18.** $\dfrac{(12+14)\cdot 2}{13\cdot 2+13\cdot 5}$ **19.** $\dfrac{4+3\div\frac{1}{7}}{2\cdot 9-3}$

Multiple Grouping Symbols

Grouping symbols include parentheses ( ), brackets [ ], braces { }, and absolute value symbols, | |. Operations within the grouping symbols are performed first. **When multiple grouping symbols occur, evaluate the expression in the innermost grouping symbols first and work outward.**

EXAMPLE 8 **Finding the Value of an Expression Containing Grouping Symbols**

Evaluate each expression:

(a) $2\cdot[3\cdot(6+3)-7]$ **(b)** $\left[4+\left(\frac{2}{3}\cdot(-9)\right)\right]\cdot 3$

Solution

(a) Perform the operation in parentheses first: $2 \cdot [3 \cdot (6 + 3) - 7] = 2 \cdot [3 \cdot 9 - 7]$

Perform the operations in brackets, multiply first: $= 2 \cdot [27 - 7]$

$= 2 \cdot [20]$

$= 40$

(b) Perform the operation in parentheses first: $\left[4 + \left(\frac{2}{3} \cdot (-9)\right)\right] \cdot 3 = [4 + (-6)] \cdot 3$

Perform the operation in brackets: $= -2 \cdot 3$

$= -6$

Quick ✓

In Problems 20 and 21, evaluate each expression.

20. $4 \cdot [2 \cdot (3 + 7) - 15]$ **21.** $2 \cdot \{4 \cdot [26 - (9 + 7)] - 15\} - 10$

When do we evaluate exponents in the order of operations? In $2 \cdot 4^3$, do we multiply first and then evaluate the exponent to obtain $2 \cdot 4^3 = 8^3 = 512$, or do we evaluate the exponent first and then multiply to obtain $2 \cdot 4^3 = 2 \cdot 64 = 128$? Because $2 \cdot 4^3 = 2 \cdot 4 \cdot 4 \cdot 4 = 128$, we **evaluate exponents before multiplication.** Don't forget to evaluate the expression in the grouping symbol first.

EXAMPLE 9 Finding the Value of an Expression Containing Exponents

Evaluate each of the following:

(a) $2 + 7(-4)^2$ **(b)** $\frac{2 \cdot 3^2 + 4}{3(2 - 6)}$

Solution

(a) Evaluate the exponent ↓

$2 + 7(-4)^2 = 2 + 7 \cdot 16$

Multiply: $= 2 + 112$

Add: $= 114$

(b) Parentheses first ↓

$\frac{2 \cdot 3^2 + 4}{3(2 - 6)} = \frac{2 \cdot 3^2 + 4}{3(-4)}$

Evaluate the exponent: $= \frac{2 \cdot 9 + 4}{3(-4)}$

Find products: $= \frac{18 + 4}{-12}$

Add terms in numerator: $= \frac{22}{-12}$

Write in lowest terms: $= \frac{2 \cdot 11}{2 \cdot -6}$

$= -\frac{11}{6}$

Quick ✓

In Problems 22–24, evaluate each of the following:

22. $\frac{7 - 5^2}{2}$ **23.** $3(7 - 3)^2$ **24.** $\frac{(-3)^2 + 7(1 - 3)}{3 \cdot 2^3 + 6}$

Order of Operations

Step 1: Perform all operations within *grouping symbols* first. When an expression has more than one set of grouping symbols, begin within the innermost grouping symbols and work outward.

Step 2: Evaluate expressions containing exponents.

Step 3: Perform *multiplication and division,* working from *left to right*.

Step 4: Perform *addition and subtraction,* working from *left to right*.

EXAMPLE 10 How to Evaluate an Expression Using Order of Operations

Evaluate: $18 + 7(2^3 - 26) + 5^2$

Step-by-Step Solution

Steps 1 and 2: Evaluate the expression in the parentheses first. In the parentheses, evaluate the expression containing the exponent first.

$18 + 7(2^3 - 26) + 5^2 = 18 + 7(8 - 26) + 25$

Evaluate 8−26 in the parentheses: $= 18 + 7(-18) + 25$

Step 3: Perform multiplication and division, working from left to right.

Multiply 7(−18): $= 18 - 126 + 25$

Step 4: Perform addition and subtraction, working from left to right.

$= -108 + 25$

$= -83$

EXAMPLE 11 Evaluating a Numerical Expression Using Order of Operations

Evaluate: $\left(\dfrac{2^3 - 6}{10 - 2 \cdot 3}\right)^2$

Solution

Evaluate the exponential expression inside the parentheses: $\left(\dfrac{2^3 - 6}{10 - 2 \cdot 3}\right)^2 = \left(\dfrac{8 - 6}{10 - 2 \cdot 3}\right)^2$

Multiply inside the parentheses: $= \left(\dfrac{8 - 6}{10 - 6}\right)^2$

Add/subtract inside the parentheses: $= \left(\dfrac{2}{4}\right)^2$

Simplify: $= \left(\dfrac{1}{2}\right)^2$

Evaluate the exponential expression: $= \dfrac{1}{4}$

Quick ✓

In Problems 25–28, evaluate each expression.

25. $\dfrac{(4 - 10)^2}{2^3 - 5}$

26. $-3[(-4)^2 - 5(8 - 6)]^2$

27. $\dfrac{(2.9 + 7.1)^2}{5^2 - 15}$

28. $\left(\dfrac{4^2 - 4(-3)(1)}{7 \cdot 2}\right)^2$

Work Smart: Study Skills

Selected problems in the exercise sets are in green. For extra help, worked solutions to these problems are in MyMathLab.

1.7 Exercises

MyMathLab® MathXL PRACTICE

Exercise numbers in green have complete video solutions in MyMathLab.

Problems **1–28** *are the* Quick ✓s *that follow the* EXAMPLES.

Building Skills

In Problems 29–32, write in exponential form. See Objective 1.

29. $5 \cdot 5$

30. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

31. $\left(-\frac{3}{5}\right)\cdot\left(-\frac{3}{5}\right)\cdot\left(-\frac{3}{5}\right)$

32. $(-8)(-8)(-8)$

In Problems 33–54, evaluate each exponential expression. See Objective 1.

33. 8^2

34. 4^2

35. $(-8)^2$

36. $(-4)^2$

37. 10^3

38. 2^5

39. -10^3

40. -2^5

41. $(-10)^3$

42. $(-2)^5$

43. $(1.5)^2$

44. $(0.04)^2$

45. -8^2

46. -4^2

47. -1^{20}

48. $(-1)^{19}$

49. 0^4

50. 1^6

51. $\left(-\frac{1}{2}\right)^6$

52. $\left(-\frac{3}{2}\right)^5$

53. $\left(-\frac{1}{3}\right)^3$

54. $\left(-\frac{3}{4}\right)^2$

In Problems 55–86, evaluate each expression. See Objective 2.

55. $2 + 3 \cdot 4$

56. $12 + 8 \cdot 3$

57. $-5 \cdot 3 + 12$

58. $-3 \cdot 12 + 9$

59. $100 \div 2 \cdot 50$

60. $50 \div 5 \cdot 4$

61. $156 - 3 \cdot 2 + 10$

62. $86 - 4 \cdot 3 + 6$

63. $(2 + 3) \cdot 4$

64. $(7 - 5) \cdot \frac{5}{2}$

65. $8 \div 4 \cdot 2$

66. $4 \div 7 \cdot 21$

67. $\frac{4 + 2}{2 + 8}$

68. $\frac{5 + 3}{3 + 15}$

69. $\frac{14 - 6}{6 - 14}$

70. $\frac{15 - 7}{7 - 15}$

71. $13 - [3 + (-8)4]$

72. $12 - [7 + (-6)3]$

73. $(-8.75 - 1.25) \div (-2)$

74. $(-11.8 - 15.2) \div (-2)$

75. $4 - 2^3$

76. $10 - 4^2$

77. $15 + 4 \cdot 5^2$

78. $10 + 3 \cdot 2^4$

79. $-2^3 + 3^2 \div (2^2 - 1)$

80. $-5^2 + 3^2 \div (3^2 + 9)$

81. $\left(\frac{4^2 - 3}{12 - 2 \cdot 5}\right)^2$

82. $\left(\frac{7 - 5^2}{8 + 4 \cdot 2}\right)^2$

83. $-2 \cdot [5 \cdot (9 - 3) - 3 \cdot 6]$

84. $3 \cdot [6 \cdot (5 - 2) - 2 \cdot 5]$

85. $\left(\frac{4}{3} + \frac{5}{6}\right)\left(\frac{2}{5} - \frac{9}{10}\right)$

86. $\left(\frac{3}{4} + \frac{1}{2}\right)\left(\frac{2}{3} - \frac{1}{2}\right)$

Mixed Practice

In Problems 87–110, evaluate each expression.

87. $4^2 - 3 \cdot 4 + 7$

88. $(-2)^2 + 4 \cdot (-2) + 11$

89. $4 + 2 \cdot (6 - 2)$

90. $3 + 6 \cdot (9 - 5)$

91. $\frac{12 - 16 \div 4 + (-24)}{16 \cdot 2 - 4 \cdot 0}$

92. $\frac{6 + 15 \div 3 + 16}{6 + 10 \cdot 0}$

93. $\left(\frac{2 - (-4)^3}{5^2 - 7 \cdot 2}\right)^2$

94. $\left(\frac{9 \cdot 2 - (-2)^3}{4^2 + 3(-1)^5}\right)^2$

95. $\frac{5^2 - 10}{3^2 + 6}$

96. $\frac{12(2)^3}{4^2 + 4 \cdot 5}$

97. $|6 \cdot (5 - 3^2)|$

98. $-6 \cdot (2 + |2 \cdot 3 - 4^2|)$

99. $\frac{4 - (-6)}{1 - (-1)}$

100. $\frac{-2 - 12}{3 - (-4)}$

101. $-\frac{7}{20} + \frac{3}{8} \div \frac{1}{2}$

102. $-\frac{4}{5} + \frac{3}{10} \div \frac{2}{9}$

103. $\frac{21 - 3^2}{1 + 3}$

104. $\frac{5 + 3^2}{2 + 5}$

105. $\frac{3}{4} \cdot \left[\frac{5}{4} \div \left(\frac{3}{8} - \frac{1}{8}\right) - 3\right]$

106. $\left[\frac{9}{10} \div \left(\frac{2}{5} + \frac{1}{5}\right) + \frac{7}{2}\right] \cdot \frac{1}{10}$

107. $\left(\frac{4}{3}\right)^3 - \left(\frac{1}{2}\right)^2 \cdot \left(\frac{8}{3}\right) + 2 \div 3$

108. $\frac{1}{18} \cdot \frac{46}{5} - \left(\frac{2}{3}\right)^2$

109. $\frac{5^2 - 3^3}{|4 - 4^2|}$

110. $\frac{3 \cdot 2^3 - 2^2 \cdot 12}{3 + 3^2}$